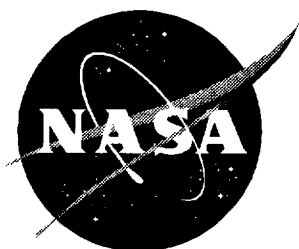


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# Application of Model Based Parameter Estimation for RCS Frequency Response Calculations Using Method of Moments

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# CONTENTS

	Abstract	2
	List of Symbols	3
1.	Introduction	6
2.	MBPE Implementation for MoM	7
3.	Numerical Results	15
4.	Concluding Remarks	18
	References	19

## **Abstract**

An implementation of the Model Based Parameter Estimation (MBPE) technique is presented for obtaining the frequency response of the Radar Cross Section (RCS) of arbitrarily shaped, three-dimensional perfect electric conductor (PEC) bodies. An Electric Field Integral Equation (EFIE) is solved using the Method of Moments (MoM) to compute the RCS. The electric current is expanded in a rational function and the coefficients of the rational function are obtained using the frequency derivatives of the EFIE. Using the rational function, the electric current on the PEC body is obtained over a frequency band. Using the electric current at different frequencies, RCS of the PEC body is obtained over a wide frequency band. Numerical results for a square plate, a cube, and a sphere are presented over a bandwidth. Good agreement between MBPE and the exact solution over the bandwidth is observed.

## List of Symbols

$\alpha$	Polarization angle of the incident electric field
$\nabla$	Del operator
$\eta_o$	Free space intrinsic impedance $377\Omega$
$\hat{\theta}$	Unit normal vector along $\theta$ direction
$\hat{\phi}$	Unit normal vector along $\phi$ direction
$(\theta_i, \phi_i)$	Incident angles
$\sigma$	Radar Cross Section
$A$	Area of the triangle
$a_n$	Coefficients of the numerator of the rational function $(n = 0, 1, 2, 3, \dots, L)$
$b_m$	Coefficients of the denominator of the rational function $(m = 0, 1, 2, 3, \dots, M)$
$C_{r,s}$	Binomial coefficient
$ds$	Surface integration with respect to observation coordinates
$ds'$	Surface integration with respect to source coordinates
EFIE	Electric Field Integral Equation
$\mathbf{E}_{inc}$	Incident electric field
$\mathbf{E}_i$	Amplitude of the incident electric field
$\mathbf{E}_{fscat}$	Scattered electric far field
$\mathbf{E}_{scat}$	Scattered electric field

$E_{xi}$	x component of $\mathbf{E}_i$
$E_{yi}$	y component of $\mathbf{E}_i$
$E_{zi}$	z component of $\mathbf{E}_i$
$f$	Frequency
$I(k)$	Unknown coefficient vector as a function of $k$
$I_i$	Unknown current coefficients
$I_n^{(m)}$	$m^{\text{th}}$ derivative of $I(k)$ w.r.t. $k$ evaluated at $n$ th frequency ( $n=1$ or $2$ )
$\mathbf{J}$	Electric current distribution
$j$	$\sqrt{-1}$
$k$	Wavenumber at any frequency $f$
$k_o$	Wavenumber at frequency $f_o$
$k_x$	As defined in equation (10)
$k_y$	As defined in equation (11)
$k_z$	As defined in equation (12)
MoM	Method of Moments
$n!$	Factorial of number $n$
PEC	Perfect Electric Conductor
$P_L(k)$	Polynomial of order $L$
$Q_M(k)$	Polynomial of order $M$
$R$	Distance between the source point and the observation point

$\mathbf{r}_i$	Coordinate of the node opposite to the edge $i$
$\mathbf{r}$	Coordinate of point in the triangle
$\mathbf{T}$	Vector testing function
$V(k)$	Excitation vector as function of $k$
$V^{(n)}(k_o)$	$n$ th derivative of $V(k)$ with respect to $k$ ; $\frac{d^n}{dk^n}V(k)$ , evaluated at $k_o$
$\mathbf{x}$	Unit normal along X-axis
$\mathbf{y}$	Unit normal along Y-axis
$Z(k)$	Impedance matrix as a function of $k$
$Z^{-1}(.)$	Inverse of the matrix $Z(.)$
$Z^{(q)}(k_o)$	$q^{\text{th}}$ derivative of $Z(k)$ with respect to $k$ ; $\frac{d^q}{dk^q}Z(k)$ , evaluated at $k_o$
$\mathbf{z}$	Unit normal along Z-axis

# 1. Introduction

The Method of Moments (MoM) using the Electric Field Integral Equation (EFIE) has been a very useful tool for accurately predicting the Radar Cross Section (RCS) of arbitrarily shaped three dimensional PEC objects [1]. To obtain the frequency response of RCS using MoM, one has to repeat the calculations over the frequency band of interest. If RCS is highly frequency dependent, one needs to do the calculations at fine increments to get an accurate representation of the frequency response. For electrically large objects, this can be computationally intensive despite the increased power of the present generation of computers. Previously, Asymptotic Waveform Evaluation (AWE) technique was applied to frequency domain electromagnetics [2,3,4]. In AWE, the unknown current is expanded in a Taylor series around a frequency. The coefficients of the Taylor series were evaluated using the frequency derivatives of EFIE. From the Taylor series, the electric current distribution on PEC body was obtained and used to calculate the RCS.

In this report, a similar but more flexible method called Model Based Parameter Estimation (MBPE) [5,6] is applied for predicting RCS of the three dimensional PEC objects over a wide band of frequencies using Method of Moments. In MBPE technique, the electric current is expanded as a rational function. The coefficients of the rational function are obtained using the frequency data and the frequency derivative data. Once the coefficients of the rational function are obtained the electric current distribution on the PEC body can be obtained at any frequency within the frequency range. Using the current distribution, the RCS is obtained. If the frequency derivative information is known for more than one frequency, a rational function matching all the samples can be obtained resulting in a wider frequency response.



The rest of the report is organized as follows. In section 2, MBPE implementation for EFIE is described. Numerical results for a square plate, a cube and a sphere are presented in section 3. The numerical data are compared with exact solution over the bandwidth. CPU time and storage requirements are given for each example. Concluding remarks on the advantages and disadvantages of MBPE are given in section 4.

## 2. MBPE Implementation for MoM

Consider an arbitrarily shaped PEC body shown in Figure 1. For RCS calculations, a plane wave is assumed to be incident at an angle  $(\theta_i, \phi_i)$ . At the surface of the PEC body the total tangential electric field is zero. The total tangential field in terms of the scattered and incident fields on the PEC body is therefore written as

$$\mathbf{E}_{scat} + \mathbf{E}_{inc} = 0 \quad (1)$$

In a subdomains MoM approach, the PEC surface is divided into triangles, rectangles, or quadrilaterals. In this paper we follow the triangular subdomain approach reported in [7]. Writing  $\mathbf{E}_{scat}$  in terms of the equivalent electric current distribution  $\mathbf{J}$  on the surface of the PEC object and applying the Galerkin's method, a set of simultaneous equations are generated and are written in a matrix equation form as

$$Z(k)I(k) = V(k) \quad (2)$$

where

$$Z(k) = \frac{jk\eta_o}{4\pi} \iint \mathbf{T} \cdot \iint \mathbf{J} \frac{\exp(-jkR)}{R} ds' ds$$

$$- \frac{j\eta_o}{4\pi k} \iint (\nabla \cdot \mathbf{T}) \iint (\nabla' \cdot \mathbf{J}) \frac{\exp(-jkR)}{R} ds' ds \quad (3)$$

and

$$V(k) = \iint \mathbf{T} \bullet \mathbf{E}_{inc} ds \quad (4)$$

$\mathbf{T}$  is the vector testing function,  $k$  is the wavenumber at frequency  $f$ , and  $\eta_o$  is the intrinsic wave impedance.  $R$  is the distance between the source point and the observation point.  $\nabla'$  indicates the del operation over the source coordinates and similarly  $ds'$  indicates the surface integration over the source coordinates. In equation (2),  $Z(k)$  is a complex and dense matrix.  $V(k)$  is the excitation column vector. Equation (4) is calculated using a harmonic plane wave

$$\mathbf{E}_{inc} = \mathbf{E}_i \exp[j(k_x x + k_y y + k_z z)] \quad (5)$$

where

$$\mathbf{E}_i = x E_{xi} + y E_{yi} + z E_{zi} \quad (6)$$

and

$$E_{xi} = \cos \theta_i \cos \phi_i \cos \alpha - \sin \phi_i \sin \alpha \quad (7)$$

$$E_{yi} = \cos \theta_i \sin \phi_i \cos \alpha + \cos \phi_i \sin \alpha \quad (8)$$

$$E_{zi} = -\sin \theta_i \cos \alpha \quad (9)$$

$$k_x = k \sin \theta_i \cos \phi_i \quad (10)$$

$$k_y = k \sin \theta_i \sin \phi_i \quad (11)$$

$$k_z = k \cos \theta_i \quad (12)$$

$\alpha$  represents the polarization angle of the incident field. When  $\alpha = 0$ , the incident field

corresponds to H-Polarization and when  $\alpha = \pi/2$ , then the incident field corresponds to E-Polarization. The matrix equation (2) is solved at any specific frequency  $f_o$  (with wavenumber  $k_o$ ) either by a direct method or an iterative method. The solution of equation (2) gives the unknown current distribution, which is used to obtain the scattered electric field. The radar cross section is given by

$$\sigma = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|\mathbf{E}_{fscat}(\mathbf{r})|^2}{|\mathbf{E}_{inc}(\mathbf{r})|^2} \quad (13)$$

The RCS given in equation (13) is calculated at one frequency. If one needs RCS over a frequency range, this calculation is to be repeated at different frequency values. Instead MBPE can be applied for rapid calculation of RCS over a frequency range. MBPE technique involves expanding the unknown coefficient vector as a rational function. The coefficients of the rational function are obtained by matching the function and its frequency derivatives of the function at one or more frequency points.

The solution of equation (2) at any frequency  $f_o$  gives the unknown current coefficient column vector  $I(k_o)$ , where  $k_o$  is the free space wavenumber at  $f_o$ . Instead  $I(k)$  can be written as a rational function,

$$I(k) = \frac{P_L(k)}{Q_M(k)} \quad (14)$$

where

$$P_L(k) = a_o + a_1k + a_2k^2 + a_3k^3 + \dots + a_Lk^L$$

$$Q_L(k) = b_o + b_1k + b_2k^2 + b_3k^3 + \dots + b_Mk^M$$

$b_o$  is set to 1 as the rational function can be divided by an arbitrary constant. The coefficients of the rational function are obtained by matching the frequency derivatives of  $I(k)$ . If equation (14) is differentiated  $t$  times with respect to  $k$ , the resulting equations can be written as[6]

$$IQ_M = P_L$$

$$I'Q_M + IQ_M' = P_L'$$

$$I''Q_M + 2I'Q_M' + IQ_M'' = P_L''$$

$$I'''Q_M + 3I''Q_M' + 3I'Q_M'' + IQ_M''' = P_L'''$$

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$$I^{(t)}Q_M + tI^{(t-1)}Q_M^{(1)} + \dots + C_{t,t-m}I^{(m)}Q^{(t-m)} + \dots + IQ_M^{(t)} = P_L^{(t)}$$

where  $C_{r,s} = \frac{r!}{s!(r-s)!}$  is the binomial coefficient. The system of  $(t+1)$  equations provides the information from which the rational function coefficients can be found if  $t \geq L + M + 1$ . If the frequency derivatives are available at only one frequency  $f_o$ , the variable in the rational function can be replaced with  $(k - k_o)$  i.e.,

$$I(k) = \frac{P_L(k - k_o)}{Q_M(k - k_o)} \quad (15)$$

and the derivatives can be evaluated at  $k = k_o$ . The coefficients of the rational function can be obtained from the following equations:

$$a_o = I(k_o) \quad (16)$$

$$\begin{bmatrix}
1 & \dots & -I_o & 0 & \dots & 0 \\
0 & \dots & -I_1 & -I_o & \dots & 0 \\
0 & \dots & -I_2 & -I_1 & \dots & 0 \\
0 & \dots & -I_3 & -I_2 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \dots & -I_{L+M-1} & I_{L+M-2} & \dots & -I_L
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_L \\
\vdots \\
b_M
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_L \\
\vdots \\
I_{L+M}
\end{bmatrix}
\quad (17)$$

where  $I_m = \frac{I^{(m)}}{m!}$ . For example for a rational function with  $L=5$  and  $M=4$ , the matrix equation

can be written as

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -I_o & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -I_1 & -I_o & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -I_2 & -I_1 & -I_o & 0 \\
0 & 0 & 0 & 1 & 0 & -I_3 & -I_2 & -I_1 & -I_o \\
0 & 0 & 0 & 0 & 1 & -I_4 & -I_3 & -I_2 & -I_1 \\
0 & 0 & 0 & 0 & 0 & -I_5 & -I_4 & -I_3 & -I_2 \\
0 & 0 & 0 & 0 & 0 & -I_6 & -I_5 & -I_4 & -I_3 \\
0 & 0 & 0 & 0 & 0 & -I_7 & -I_6 & -I_5 & -I_4 \\
0 & 0 & 0 & 0 & 0 & -I_8 & -I_7 & -I_6 & -I_5
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
b_1 \\
b_2 \\
b_3 \\
b_4
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
I_7 \\
I_8 \\
I_9
\end{bmatrix}
\quad (18)$$

This approach is same as the Padé approximation given in [8]. This method has been successfully applied to electromagnetic scattering from cavity-backed apertures using a hybrid finite element and method of moments technique[9].

If the frequency derivatives are known at more than one frequency, then the expansion about  $k=k_o$  cannot be used and the system matrix to solve the rational function coefficients takes a general form. For the sake of simplicity, only a two frequency model is presented here. Assume that at two frequencies,  $f_1$  (with free space wavenumber  $k_1$ ) and  $f_2$  (with free space wavenumber

$k_2$ ), four derivatives are evaluated at each frequency. Hence 10 samples of data are available (2 frequency samples and a total of 8 frequency derivative samples) to form a rational function with  $L=5$  and  $M=4$

$$I(k) = \frac{a_o + a_1k + a_2k^2 + a_3k^3 + a_4k^4 + a_5k^5}{1 + b_1k + b_2k^2 + b_3k^3 + b_4k^4} \quad (19)$$

Equation (19) can be written as

$$(1 + b_1k + b_2k^2 + b_3k^3 + b_4k^4)I(k) = a_o + a_1k + a_2k^2 + a_3k^3 + a_4k^4 + a_5k^5 \quad (20)$$

Differentiating equation (20) four times at each frequency, the matrix equation for the solution of the coefficients of the rational function (equation (19)) can be written as

$$\begin{bmatrix} 1 & k_1 & k_1^2 & k_1^3 & k_1^4 & k_1^5 & I_1^{(0)}k_1 & I_1^{(0)}k_1^2 & I_1^{(0)}k_1^3 & I_1^{(0)}k_1^4 \\ 0 & 1 & 2k_1 & 3k_1^2 & 4k_1^3 & 5k_1^4 & M_{27} & M_{28} & M_{29} & M_{2,10} \\ 0 & 0 & 2 & 6k_1 & 12k_1^2 & 20k_1^3 & M_{37} & M_{38} & M_{39} & M_{3,10} \\ 0 & 0 & 0 & 6 & 24k_1 & 60k_1^2 & M_{47} & M_{48} & M_{49} & M_{4,10} \\ 0 & 0 & 0 & 0 & 24 & 120k_1 & M_{57} & M_{58} & M_{59} & M_{5,10} \\ 1 & k_2 & k_2^2 & k_2^3 & k_2^4 & k_2^5 & I_2^{(0)}k_2 & I_2^{(0)}k_2^2 & I_2^{(0)}k_2^3 & I_2^{(0)}k_2^4 \\ 0 & 1 & 2k_2 & 3k_2^2 & 4k_2^3 & 5k_2^4 & M_{77} & M_{78} & M_{79} & M_{7,10} \\ 0 & 0 & 2 & 6k_2 & 12k_2^2 & 20k_2^3 & M_{87} & M_{88} & M_{89} & M_{8,10} \\ 0 & 0 & 0 & 6 & 24k_2 & 60k_2^2 & M_{97} & M_{98} & M_{99} & M_{9,10} \\ 0 & 0 & 0 & 0 & 24 & 120k_2 & M_{10,7} & M_{10,8} & M_{10,9} & M_{10,10} \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} I_1^{(0)} \\ I_1^{(1)} \\ I_1^{(2)} \\ I_1^{(3)} \\ I_1^{(4)} \\ I_2^{(0)} \\ I_2^{(1)} \\ I_2^{(2)} \\ I_2^{(3)} \\ I_2^{(4)} \end{bmatrix} \quad (21)$$

where  $I_1^{(m)} = \frac{d^m}{dk^m} I(k)|_{k=k_1}$ ,  $I_2^{(m)} = \frac{d^m}{dk^m} I(k)|_{k=k_2}$  and

$$M_{27} = -(I_1^{(1)}k_1 + I_1^{(0)})$$

$$M_{28} = -(I_1^{(1)}k_1^2 + 2I_1^{(0)}k_1)$$

$$M_{29} = -(I_1^{(1)}k_1^3 + 3I_1^{(0)}k_1^2)$$

$$M_{2 \ 10} = -(I_1^{(1)}k_1^4 + 4I_1^{(0)}k_1^3)$$

$$M_{37} = -(I_1^{(2)}k_1 + 2I_1^{(1)})$$

$$M_{38} = -(I_1^{(2)}k_1^2 + 4I_1^{(1)}k_1 + 2I_1^{(0)})$$

$$M_{39} = -(I_1^{(2)}k_1^3 + 6I_1^{(1)}k_1^2 + 6I_1^{(0)}k_1)$$

$$M_{3 \ 10} = -(I_1^{(2)}k_1^4 + 8I_1^{(1)}k_1^3 + 12I_1^{(0)}k_1^2)$$

$$M_{47} = -(I_1^{(3)}k_1 + 3I_1^{(2)})$$

$$M_{48} = -(I_1^{(3)}k_1^2 + 6I_1^{(2)}k_1 + 6I_1^{(1)})$$

$$M_{49} = -(I_1^{(3)}k_1^3 + 9I_1^{(2)}k_1^2 + 18I_1^{(1)}k_1 + 6I_1^{(0)})$$

$$M_{4 \ 10} = -(I_1^{(3)}k_1^4 + 12I_1^{(2)}k_1^3 + 36I_1^{(1)}k_1^2 + 24I_1^{(0)}k_1)$$

$$M_{57} = -(I_1^{(4)}k_1 + 4I_1^{(3)})$$

$$M_{58} = -(I_1^{(4)}k_1^2 + 8I_1^{(3)}k_1 + 12I_1^{(2)})$$

$$M_{59} = -(I_1^{(4)}k_1^3 + 12I_1^{(3)}k_1^2 + 36I_1^{(2)}k_1 + 24I_1^{(0)})$$

$$M_{5 \ 10} = -(I_1^{(4)}k_1^4 + 16I_1^{(3)}k_1^3 + 72I_1^{(2)}k_1^2 + 96I_1^{(1)}k_1 + 24I_1^{(0)})$$

$$M_{77} = -(I_2^{(1)}k_2 + I_2^{(0)})$$

$$M_{78} = -(I_2^{(1)}k_2^2 + 2I_2^{(0)}k_2)$$

$$M_{78} = -(I_2^{(1)}k_2^3 + 3I_2^{(0)}k_2^2)$$

$$M_{7\ 10} = -(I_2^{(1)}k_2^4 + 4I_2^{(0)}k_2^3)$$

$$M_{87} = -(I_2^{(2)}k_2 + 2I_2^{(1)})$$

$$M_{88} = -(I_2^{(2)}k_2^2 + 4I_2^{(1)}k_2 + 2I_2^{(0)})$$

$$M_{89} = -(I_2^{(2)}k_2^3 + 6I_2^{(1)}k_2^2 + 6I_2^{(0)}k_2)$$

$$M_{8\ 10} = -(I_2^{(2)}k_2^4 + 8I_2^{(1)}k_2^3 + 12I_2^{(0)}k_2^2)$$

$$M_{97} = -(I_2^{(3)}k_2 + 3I_2^{(2)})$$

$$M_{98} = -(I_2^{(3)}k_2^2 + 6I_2^{(2)}k_2 + 6I_2^{(1)})$$

$$M_{99} = -(I_2^{(3)}k_2^3 + 9I_2^{(2)}k_2^2 + 18I_2^{(1)}k_2 + 6I_2^{(0)})$$

$$M_{9\ 10} = -(I_2^{(3)}k_2^4 + 12I_2^{(2)}k_2^3 + 36I_2^{(1)}k_2^2 + 24I_2^{(0)}k_2)$$

$$M_{10\ 7} = -(I_2^{(4)}k_2 + 4I_2^{(3)})$$

$$M_{10\ 8} = -(I_2^{(4)}k_2^2 + 8I_2^{(3)}k_2 + 12I_2^{(2)})$$

$$M_{10\ 9} = -(I_2^{(4)}k_2^3 + 12I_2^{(3)}k_2^2 + 36I_2^{(2)}k_2 + 24I_2^{(0)})$$

$$M_{10\ 10} = -(I_2^{(4)}k_2^4 + 16I_2^{(3)}k_2^3 + 72I_2^{(2)}k_2^2 + 96I_2^{(1)}k_2 + 24I_2^{(0)})$$



In the above equations,  $I^{(t)}$ , the  $t^{\text{th}}$  derivative can be obtained using the recursive relationship,

$$I^{(t)}(k) = Z^{-1}(k) \left[ V^{(t)} - \sum_{q=0}^t C_{t,q} Z^{(q)}(k) I^{(t-q)}(k) \right] \quad (22)$$

$Z^{(q)}(k)$  is the  $q^{\text{th}}$  derivative of  $Z(k)$  with respect to  $k$ . Similarly  $V^{(t)}(k)$  is the  $t^{\text{th}}$  derivative of  $V(k)$  with respect to  $k$ .  $C_{t,q}$  is the binomial coefficient.

The above procedure can be generalized for multiple frequencies with frequency derivatives evaluated at each frequency to increase the accuracy of the rational function. Alternatively, the two-frequency-four-derivative model can be used with multiple frequency windows. As the complexity of the matrix equation to solve for multiple-frequency-multiple derivative model increase with number of frequency points and number of derivatives taken at each frequency, the two-frequency-four-derivative model is followed in this report.

### 3. Numerical Results

To validate the analysis presented in the previous sections, a few numerical examples are considered. RCS frequency response calculations are done for a square plate, a cube, and a sphere. The numerical data obtained using MBPE are compared with the results calculated at each frequency using the triangular patch Method of Moments. We will refer to the latter method as “exact solution.” All the computations reported below are done on a SGI *Indigo 2* (with IP 22 processor) computer.

#### (a) Square Plate:

The first example is a square plate ( $1\text{cm} \times 1\text{cm}$ ) with the incident electric field at

$\theta_i = 90^\circ$  and  $\phi_i = 0^\circ$ . The incident field is E-polarized ( $\alpha = 90^\circ$ ). The square plate is discretized with 603 unknowns. The frequency response is calculated with one-frequency MBPE ( $L=5$ , and  $M=4$ ) at 30GHz and using nine frequency derivatives at that frequency. The frequency response is also calculated with a two-frequency MBPE ( $L=5$ ,  $M=4$ ) at  $f_1=25\text{GHz}$  and  $f_2=35\text{GHz}$  and using four frequency derivatives at each frequency. The frequency responses obtained are plotted in figure 2 along with the exact solution calculated at each frequency over the frequency range 15GHz to 45GHz. The one-frequency MBPE took 1688 secs to generate the moments, whereas two-frequency MBPE took a total of 3060 secs to generate moments at both frequencies. The exact solution took 22,258 secs to calculate 31 frequency values from 15GHz to 45GHz. It can be seen that both one-frequency MBPE and two-frequency MBPE give accurate results over the frequency range 15GHz to 45GHz. One-frequency MBPE seems to compute the results much faster than the exact solution and two-frequency MBPE.

**(b) Cube:**

RCS frequency response of a PEC cube ( $1\text{cm} \times 1\text{cm} \times 1\text{cm}$ ) is computed for normal incidence. One-frequency MBPE with  $L=5$  and  $M=4$  at  $f_0=15\text{GHz}$  is used to calculate the frequency response. Frequency response is also calculated using the two-frequency MBPE with  $L=5$  and  $M=4$  at  $f_1=11\text{GHz}$  and  $f_2=19\text{GHz}$ . The frequency responses obtained are plotted in Figure 3 along with the exact solution calculated at each frequency over the frequency range 2GHz to 22GHz. The one-frequency MBPE took 1143 secs of CPU time to generate the moments, whereas the two-frequency MBPE took a total of 2066 secs to generate the moments at both frequencies. The exact solution took 10,500 secs to calculate RCS at 21 frequency values from 2GHz to 22GHz. It can be seen from Figure 3 that the one-frequency MBPE gives accurate

solution over the frequency range 2GHz to 22GHz, whereas the two-frequency MBPE gives accurate solution over the frequency range 5GHz to 20GHz. It can also be seen that one-frequency MBPE is faster than the two-frequency MBPE and exact solution.

### (c) Sphere

As a third example, a PEC sphere of radius 0.318cm is considered. The sphere is discretized into 248 triangular elements. One frequency MBPE with  $L=5$  and  $M=4$  at  $f_o=20\text{GHz}$  is used to calculate the frequency response. Two-frequency MBPE with  $L=5$  and  $M=4$  is at  $f_1=15\text{GHz}$  and  $f_2=25\text{GHz}$  is also used to calculate the frequency response. The frequency response over the frequency range is plotted in Figure 4 along with the exact solution calculated with 1GHz frequency interval over the bandwidth. The one-frequency MBPE took 580 secs to generate the moments, whereas two-frequency MBPE took a total of 1040 secs to generate the moments at both frequencies. The exact solution took 7905 secs to calculate RCS at 31 frequency values from 5GHz to 35GHz. It can be seen from Figure 4 that the one-frequency MBPE and two-frequency MBPE gives accurate solution over the frequency

**Comment on Storage:** In all the above examples, when solving a matrix equation, one needs to store a complex, dense matrix  $Z(k_o)$  of size  $N \times N$  for exact solution at each frequency. In one-frequency MBPE one needs to store  $(L+M)$  complex dense matrices  $(Z^{(q)}(k_o))$ ,  $q=1,2,3,\dots,(L+M)$  of size  $N \times N$ , along with the matrix  $Z(k_o)$  of size  $N \times N$ . For electrically large problems, this could impose a burden on computer resources. This problem can be overcome by storing the derivative matrices,  $Z^{(q)}(k_o)$  out-of-core, as the derivative matrices are

required only for matrix-vector multiplication. In two-frequency MBPE, one needs to store only  $\left(\frac{L+M-1}{2}\right)$  derivative matrices of size  $N \times N$  along with the matrix  $Z(k)$  of size  $N \times N$  at each frequency. Once the moments are calculated at one frequency, the memory used for the matrices can be reutilized to generate moments at the second frequency, hence reducing the burden on computer memory requirements. In all the numerical examples presented with  $L=5$  and  $M=4$ , one-frequency MBPE had to store 10 matrices of size  $N \times N$ , whereas two-frequency MBPE had to store only 5 matrices of size  $N \times N$  at each frequency. The memory to store the matrices at one frequency is reutilized to store the matrices at the second frequency. Hence, even though the CPU timings for two-frequency MBPE is more than the one-frequency MBPE, but if computer memory is a constraint, however, it is advisable to use two-frequency MBPE as an alternative to one-frequency MBPE.

## 4. Concluding Remarks

An implementation of MBPE for frequency domain Method of Moments is presented. The RCS frequency response for different PEC objects such as a square plate, cube, and sphere are computed and compared with the exact solution. From the numerical examples presented in this report, MBPE is found to be superior in terms of the CPU time to obtain a frequency response. It may also be noted that although calculations are done at one incidence angle for all the examples presented, with a nominal cost, the frequency response at multiple incidence angles can also be calculated. It is also observed from the numerical examples that the one-frequency MBPE is superior to two-frequency MBPE in terms of the CPU timings, whereas two-frequency MBPE is superior in terms of the computer memory requirements. As MBPE results in a rational function

one can extract the poles and zeros of this function and can construct the time response, which is useful in microwave imaging applications.

In one-frequency MBPE the frequency response is valid over a certain frequency range. In two-frequency MBPE, the two frequency values have to be chosen so as to get an accurate frequency response between the two frequency values. To get a wide frequency response for any problem, either one- or two-frequency MBPE models have to be used with different frequency values to cover the complete frequency range. To be accurate over all frequency ranges a reliable error criteria should be developed, which can be used to sample the frequency points to apply MBPE model. Development of such a sampling criteria will make MBPE a very effective tool for computational electromagnetics.

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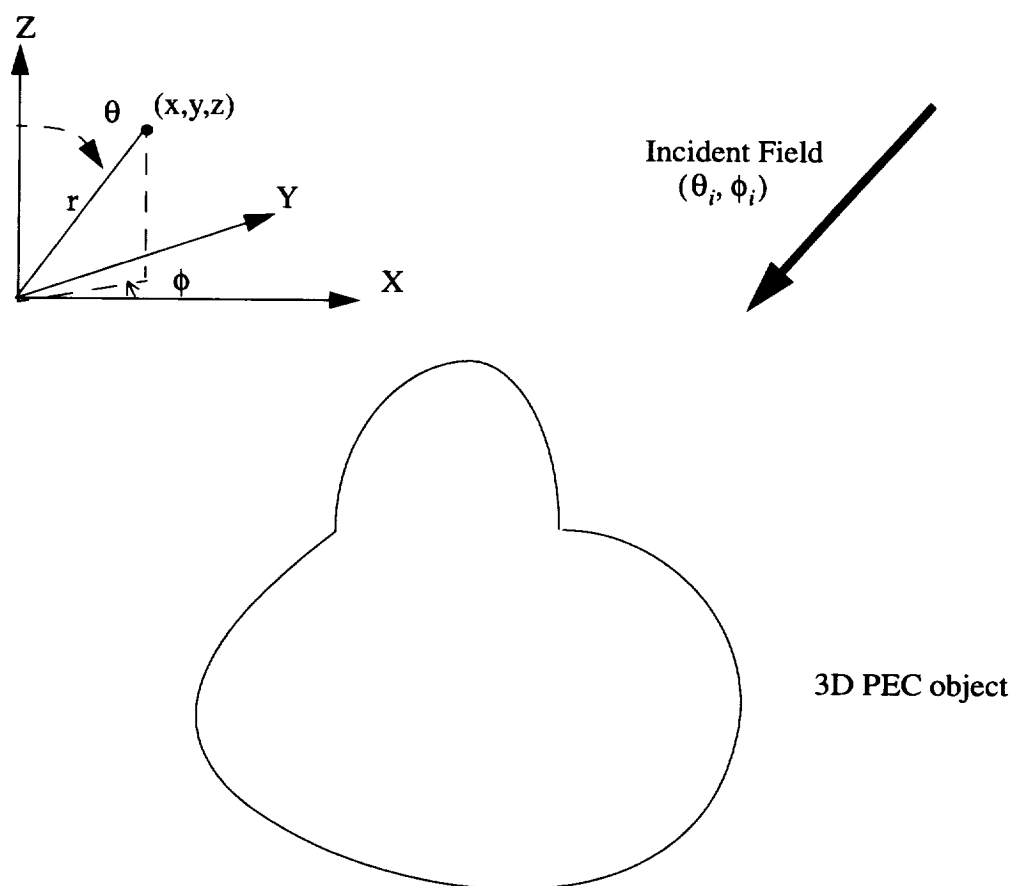


Figure 1 Arbitrarily shaped three dimensional PEC object

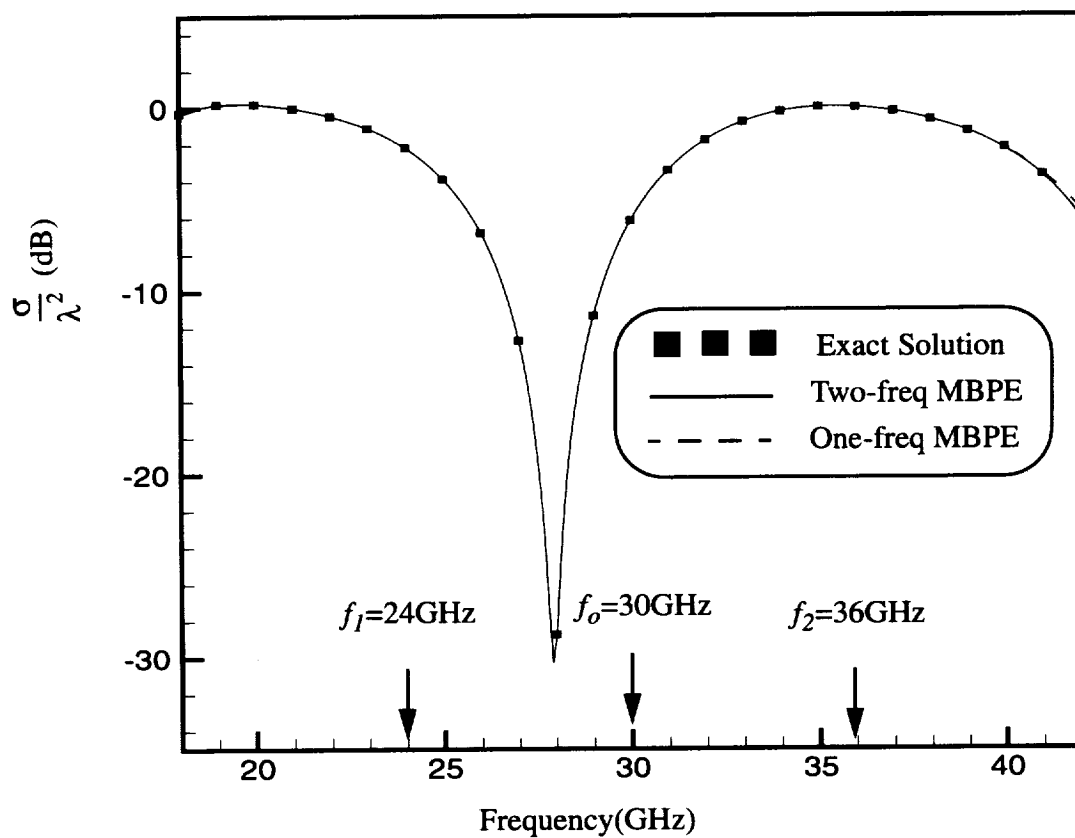


Figure 2 RCS frequency response of a square plate(1cmX1cm)



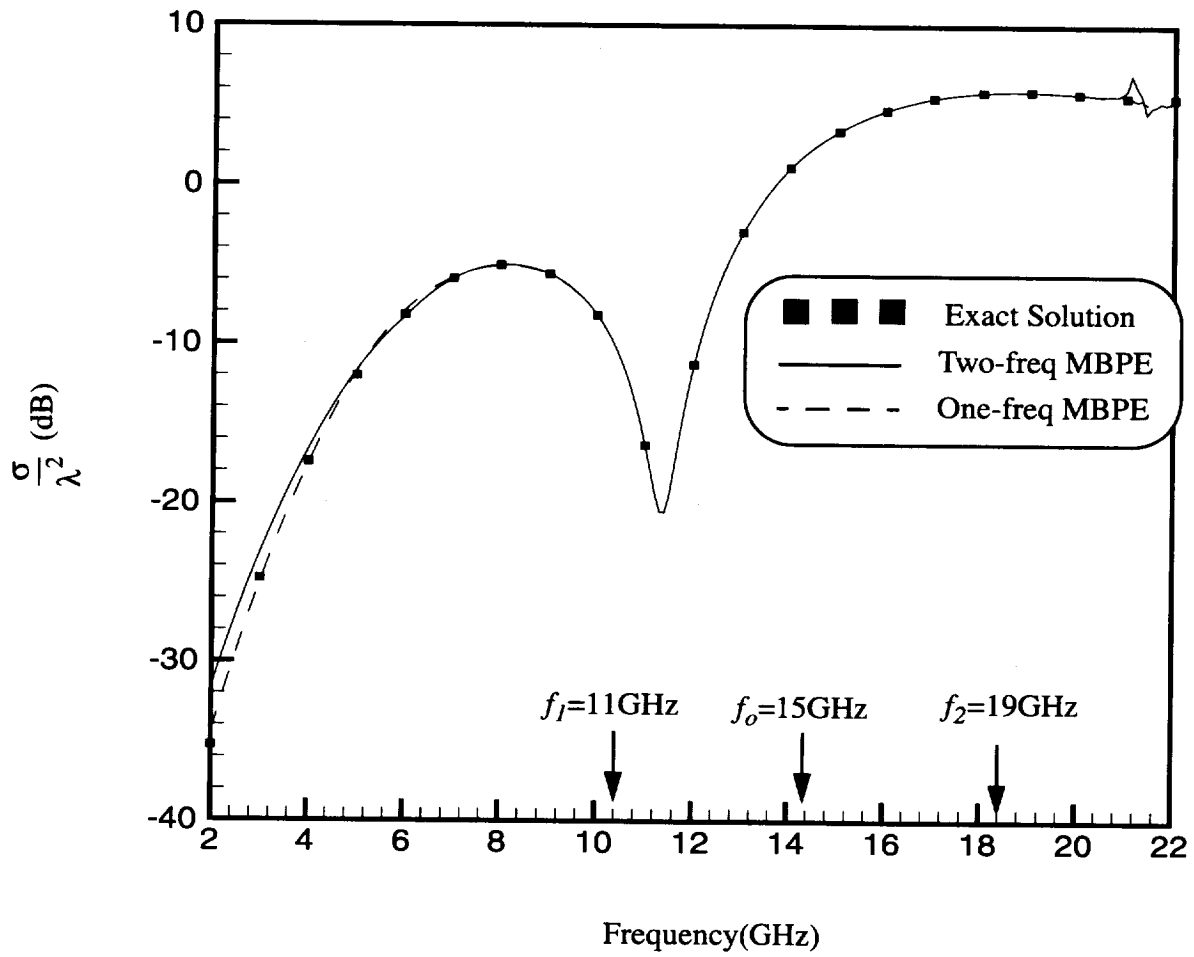


Figure 3 RCS frequency response of a PEC cube ( $1\text{cm} \times 1\text{cm} \times 1\text{cm}$ )

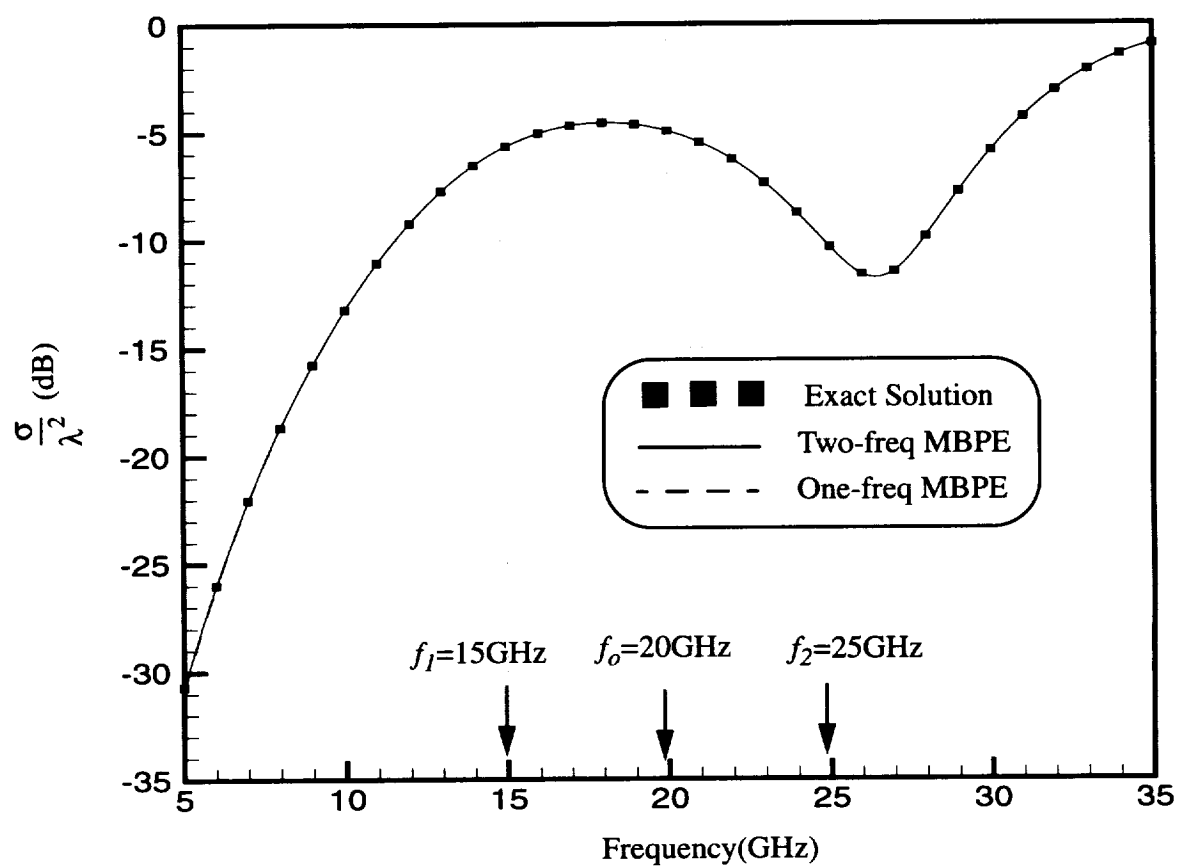


Figure 4 RCS frequency response of a PEC sphere (*radius=0.318cm*)



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13. ABSTRACT (Maximum 200 words) An implementation of the Model Based Parameter Estimation (MBPE) technique is presented for obtaining the frequency response of the Radar Cross Section (RCS) of arbitrarily shaped, three-dimensional perfect electric conductor (PEC) bodies. An Electric Field Integral Equation (EFIE) is solved using the Method of Moments (MoM) to compute the RCS. The electric current is expanded in a rational function and the coefficients of the rational function are obtained using the frequency derivatives of the EFIE. Using the rational function, the electric current on the PEC body is obtained over a frequency band. Using the electric current at different frequencies, RCS of the PEC body is obtained over a wide frequency band. Numerical results for a square plate, a cube, and a sphere are presented over a bandwidth. Good agreement between MBPE and the exact solution over the bandwidth is observed.				
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